Amortizing Floor Option Valuation

An amortizing floor option consists of 12 floorlets, or put options, on the arithmetic average of the daily 12-month Pibor rate fixings over respective windows of approximately 30 calendar days. Furthermore the notional amount corresponding to each floorlet is specified by an amortization schedule.

We consider a floor option consisting of a series of floorlets as follows. Here each floorlet is specified by

- settlement time, T ,
- set of Pibor fixing times, $\{\tau_i\}$, such that $0 < \tau_1 < ... < \tau_m < T$,
- payoff at T of the form

$$N \times \Delta \times \left(X - \frac{1}{m} \sum_i^m L_i \right)^+$$

  where

  - L_i denotes the δ -period Pibor rate that sets at τ_i ,
  - N is an FRF notional amount,
  - Δ is an accrual period.

For each period in the tables above, a party is short a floorlet specified by

- strike, 3.55%,

- underlying interest rate equal to the arithmetic average of the 12 month Pibor-rate daily fixings in the averaging window, $\frac{1}{m}\sum_{i}^{m} L_i$,
- payment at settlement date of the form

$$N \times \Delta \times \left(3.55\% - \frac{1}{m}\sum_{i}^{m} L_i\right)^{+}$$

where

  - N is a corresponding notional amount,
  - m is the number of daily fixings in the averaging window,
  - Δ is the interval of time from the accrual start date to the settlement date calculated according to the ACT/360 day-counting convention.

Consider a floorlet with associated Pibor rate averaging window, and let T denote the corresponding settlement date. We assume that, for each reset in this window, the associated 12-month forward Pibor rate, L , satisfies an SDE, of the form

$$dL = L\sigma\, dW ,$$

under the T -forward probability measure, where

- σ is a constant volatility parameter,
- W is a standard Brownian motion.

We note that, mathematically, the Pibor rates above cannot *simultaneously* be martingales under the common T -forward probability measure; moreover, in order to simultaneously express the Pibor rates above under this same measure, the SDE above requires a drift correction term.

Let $Z = \frac{1}{m}\sum_{i}^{m} L_i$ be the arithmetic average of the 12-month Pibor rates over the daily fixings in the averaging window above. Since each Pibor rate is log-normal, Z cannot also be log-normal. For computational speed, then, we approximate Z by a random variable,

$$\hat{Z} = \left(\frac{1}{m}\sum_{i}^{m} L_i^0\right) e^{-\frac{\sigma^2}{2}\hat{T} + \sigma \hat{W}_{\hat{T}}},$$

where

- $\hat{T} = \frac{1}{m}\sum_{i}^{m} T_i$,
- $\sigma = \sqrt{\frac{1}{\hat{T}}\left(\frac{1}{m}\sum_{i}^{m} \sigma_i^2 T_i\right)}$,
- $\hat{W}$ is a standard Brownian motion.

Here, with respect to the i^{th} fixing point in the averaging window,

- T_i denotes the reset time,
- σ_i is the forward Pibor rate volatility,
- L_i^0 denotes the forward Pibor rate.

From the above (see Appendix A) we see that $E(Z) = E(\hat{Z})$ and $E(Z^2) \approx E(\hat{Z}^2)$.

The random variable Z can then be viewed as a moment matching approximation to $\hat{Z}$.

We now approximates the price of a floorlet by

$$N \times \Delta \times d_T \times E^T\left[\left(3.55\% - \hat{Z}\right)^+\right]$$

where

- Δ is the interval from the accrual start date to the settlement date,
- d_T is the discount factor to the settlement date, and
- N is a Euro notional amount.

We compute, over a specified averaging window,

- the average Pibor fixing time, $\hat{T} = \frac{1}{m}\sum_i^m T_i$,
- the average forward Pibor rate, $\frac{1}{m}\sum_i^m L_i^0$,
- an average volatility, $\sqrt{\frac{1}{\hat{T}}\frac{\sum_i^m \sigma_i^2 T_i}{m}}$.

Here the forward Pibor rate volatility, σ_i , is taken from a corresponding Euro forward swap rate volatility curve. Furthermore Pibor forward rates are calculated from a curve sheet of EURIBOR discount factors (see https://finpricing.com/lib/IrBasisCurve.html).

Consider a fixed-for-floating interest rate swap with fixed payments, $\Delta_i \kappa$, at settlement time, t_i , for $i = 1,\ldots,p$, where

- κ is a fixed rate, and
- $\Delta_i = T_i - T_{i-1}$.

The addin *PV01* then calculates

$$\sum_i^p \Delta_i d_i$$

where d_i is the discount factor to time t_i. This term can be viewed as the denominator term in an associated forward swap measure numeraire process,

$$\left\{ \frac{\sum_i^p \Delta_i P(t,T_i)}{\sum_i^p \Delta_i d_i} \middle| 0 \le t \le T \right\};$$

here

- T is the swap's forward start time, and
- $P(T,T_i)$ denotes the price at time T of a zero coupon bond of maturity T_i.

With respect to our particular deal, Tranche 1, *swapPV01* is applied to compute Δd_T where

- Δ is the interval (calculated using ACT/360 day-counting) from the accrual start date to the settlement date (see Table 2.1.2), and
- d_T is the discount factor to the settlement date, T.